DIAGRAMMATIC QUANTUM CIRCUIT COMPRESSION FOR HAMILTONIAN SIMULATION

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CONTRIBUTION

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OUTLINE

- Motivation
- ZX-calculus and Pauli-gadgets
- Compression of Quantum Circuits for Hamiltonian Simulation

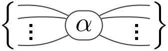
MOTIVATION

- NISQ: Restrictions on number of qubits and gates
- Shallow and simple quantum circuit structures are crucial
- Increasing popularity of diagrammatic approaches to quantum computing
- Used tool: ZX-calculus

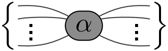
ZX-calculus and Pauli-gadgets

ZX-DIAGRAMS: GENERATORS

Spiders:

m  n

$$:= \underbrace{|0,\dots,0\rangle}_{n}\underbrace{\langle 0,\dots,0|}_{m} + e^{i\alpha}\underbrace{|1,\dots,1\rangle}_{n}\underbrace{\langle 1,\dots,1|}_{m}, \quad (1)$$

m  n

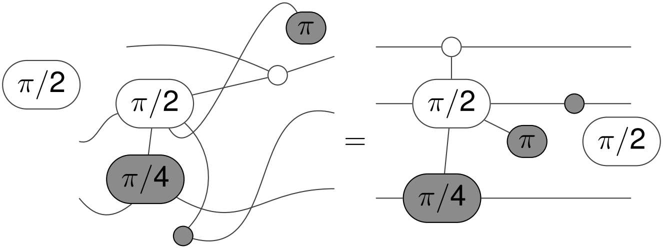
$$:= \underbrace{|+,\dots,+\rangle}_{n}\underbrace{\langle +,\dots,+|}_{m} + e^{i\alpha}\underbrace{|-,\dots,-\rangle}_{n}\underbrace{\langle -,\dots,-|}_{m} \quad (2)$$

Hadamard:

$$= 1/\sqrt{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (3)$$

[1] Van de Wetering, J.: ZX-calculus for the working quantum computer scientist, 2020, arXiv: 2012.13966 [quant-ph].

ZX-DIAGRAMS: ONLY CONNECTIVITY MATTERS



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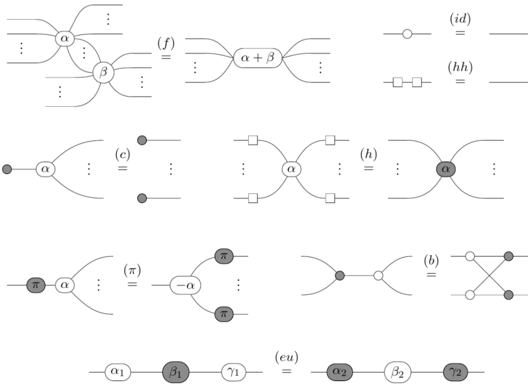
[1] Van de Wetering, J.: ZX-calculus for the working quantum computer scientist, 2020, arXiv: 2012.13966 [quant-ph].

ZX-CALCULUS



A presentation of the ZX-calculus. All rules hold for all $\alpha, \ \beta \in \mathbb{R}$ and due to (*h*) and (*hh*) for all colors interchanged. These rules only hold up to a non-zero scalar.

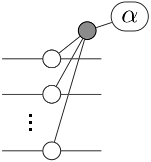
[1] Van de Wetering, J.: ZX-calculus for the working quantum computer scientist, 2020, arXiv: 2012.13966 [quant-ph].

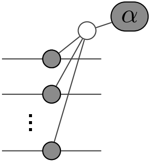
PHASE-GADGETS

The Z-phase gadgets $\Phi_n^Z(\alpha) : \mathbb{C}^{\otimes n} \longrightarrow \mathbb{C}^{\otimes n}$ are a family of unitary maps and are recursively defined as:

$$\begin{aligned} \Phi_1^Z(\alpha) &:= Z(\alpha), \\ \Phi_{n+1}^Z(\alpha) &:= (\mathbb{1}_n \otimes CNOT(n+1, n))(\Phi_n^Z \otimes \mathbb{1}_1)(\mathbb{1}_n \otimes CNOT(n+1, n)) \end{aligned} \tag{5}$$

In ZX-calculus:

$\Phi_n^Z(\alpha) =$ 

, $\Phi_n^X(\alpha) =$ 

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[2] Cowtan, A. et al.: Phase Gadget Synthesis for Shallow Circuits. Electronic Proceedings in Theoretical Computer Science 318, pp. 213–228, 2020, https://doi.org/10.4204%2Feptcs.318.13.

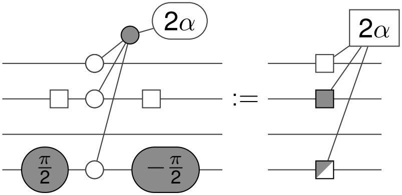
PAULI-GADGETS

Let s be a word over the alphabet $\{X, Y, Z\}$, then a Pauli-gadget $P(\alpha, s)$ is defined as $U(s)\Phi^{Z}_{|s|}(\alpha)U(s)^{\dagger}$ where the unitary $U(s)$ is defined recursively over s:

$$U(Zs') = I \otimes U(s'), \quad U(Ys') = X(\pi/2) \otimes U(s'), \quad U(Xs') = H \otimes U(s') \tag{7}$$

Example:

$$e^{-i\alpha ZXIY} = \quad (8)$$



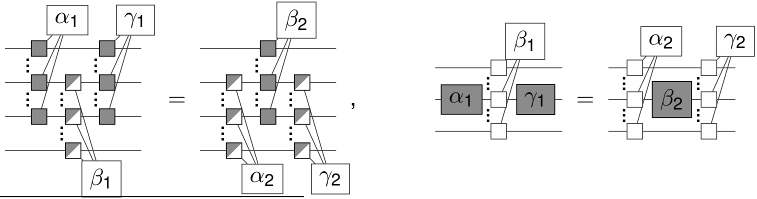
[2] Cowtan, A. et al.: Phase Gadget Synthesis for Shallow Circuits. Electronic Proceedings in Theoretical Computer Science 318, pp. 213–228, 2020, https://doi.org/10.4204%2Feptcs.318.13.

FUSION AND TURNOVER RULES

- Let *s* be a Pauli-string, then for all α and β:

$$P(\alpha, s)P(\beta, s) = P(\alpha + \beta, s) \tag{9}$$

- Two gadgets with an even number of legs on the same wires commute.

- 

[2] Cowtan, A. et al.: Phase Gadget Synthesis for Shallow Circuits. Electronic Proceedings in Theoretical Computer Science 318, pp. 213–228, 2020, https://doi.org/10.4204%2Feptcs.318.13.

[3] Winderl, D. et al.: A recursively partitioned approach to architecture- aware ZX Polynomial synthesis and optimization, 2023, arXiv: 2303.17366 [quant- ph]

Diagrammatic Compression for Hamiltonian Simulation

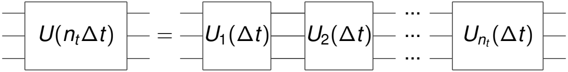
HAMILTONIAN SIMULATION

Time evolution operator:

$$U(t_0, t_1) = \tau \exp\left(-i \int_{t_0}^{t_1} H(t)dt\right), \quad H(t) = \sum_{\ell} H_{\ell}(t) \tag{10}$$

By discretization in time and Trotter decomposition:
Approximate time evolution operator by

$$U(n_t \Delta t) = \prod_{k=0}^{n_t - 1} U_{n_t - k}(\Delta t), \quad U_k(\Delta t) = \prod_{\ell} \exp(-i H_{\ell,k} \Delta t). \tag{11}$$



[5] Camps, D. et al.: An Algebraic Quantum Circuit Compression Algorithm for Hamiltonian Simulation. SIAM Journal on Matrix Analysis and Applications 43 (3), pp. 1084–1108, 2022, https://doi.org/10.1137%F21m1439298.

COMPRESSION USING TURNOVER OPERATIONS

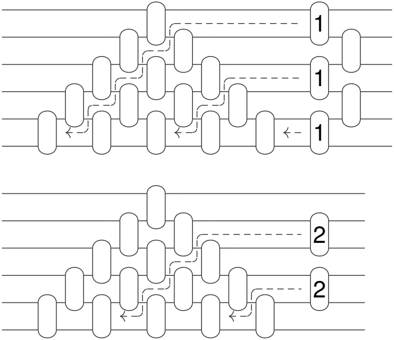
Definition

Define a *block* $B_i(\vec{\Theta})$ as a structure that satisfies

1. Fusion: $B_i(\vec{\alpha})B_i(\vec{\beta}) = B_i(\vec{a})$
2. Commutation: $B_i(\vec{\alpha})B_j(\vec{\beta}) = B_j(\vec{\beta})B_i(\vec{\alpha})$ for $\mid i - j \mid > 1$
3. Turnover: $B_i(\vec{\alpha})B_{i+1}(\vec{\beta})B_i(\vec{\gamma}) = B_{i+1}(\vec{a})B_i(\vec{b})B_{i+1}(\vec{c})$

[4] Kökcü, E. et al.: Algebraic compression of quantum circuits for Hamiltonian evolution. Physical Review A 105 (3), 2022, https://doi.org/10.1103%2Fphysreva.105.032420.

MERGING A TROTTER STEP INTO A CONSTANT-DEPTH TRIANGLE



[5] Camps, D. et al.: An Algebraic Quantum Circuit Compression Algorithm for Hamiltonian Simulation. SIAM Journal on Matrix Analysis and Applications 43 (3), pp. 1084–1108, 2022, https://doi.org/10.1137%F21m1439298.

COMPRESSION ALGORITHM

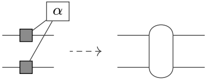
Input: Trotter circuit C on N spins with n_t time-steps, $n_t > N/2$
Output: Compressed $N \times N$ Trotter circuit C' equivalent to C

1: $C' \leftarrow \mathit{TriangleCircuit}(C'[:, 1:N])$
2: **for** $l = N+1$ to $2n_t$ **do**
3: $\quad \mathit{MergeLayer}(C', C[:, l])$
4: $C' \leftarrow \mathit{SquareCircuit}(C')$

[4] Camps, D. et al.: An Algebraic Quantum Circuit Compression Algorithm for Hamiltonian Simulation. SIAM Journal on Matrix Analysis and Applications 43 (3), pp. 1084–1108, 2022, https://doi.org/10.1137%F21m1439298.

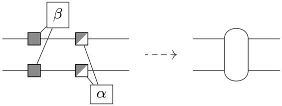
BLOCKS IN ZX-CALCULUS

- Kitaev Chain:



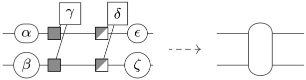
$$H(t) = \sum_{i=1}^{N-1} J_i^{\alpha_i}(t)\sigma_i^{\alpha_i}\sigma_{i+1}^{\alpha_i}, \quad \text{for } \alpha_i \in \{x, y, z\},\ \alpha_i \neq \alpha_{i+1} \tag{12}$$

- XY-Model:



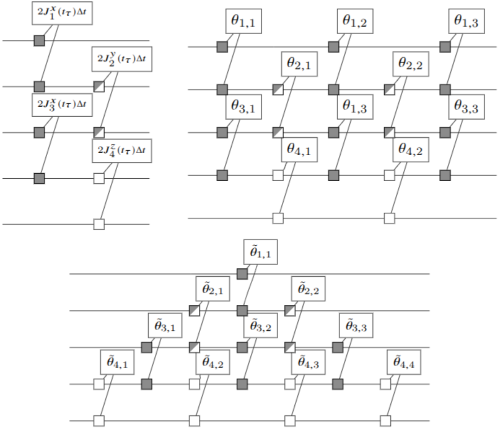
$$H(t) = \sum_{i=1}^{N-1} J_i^{\alpha}(t)\sigma_i^{\alpha}\sigma_{i+1}^{\alpha} + J_i^{\beta}(t)\sigma_i^{\beta}\sigma_{i+1}^{\beta}, \quad \text{for } \alpha \neq \beta \in \{x, y, z\} \tag{13}$$

- Transverse Field XY-Model:



$$H(t) = \sum_{i=1}^{N-1} J_i^{x}(t)\sigma_i^{x}\sigma_{i+1}^{x} + J_i^{y}(t)\sigma_i^{y}\sigma_{i+1}^{y} + \sum_{i=1}^{N} h_i^{z}(t)\sigma_i^{z} \tag{14}$$

KITAEV-COMPRESSION FOR 5 QUBITS



One Trotter timestep (top left). The constant-depth square circuit for 5 qubits (top right).

The constant-depth triangle circuit for 5 qubits (bottom).

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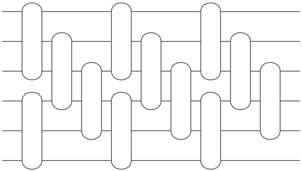
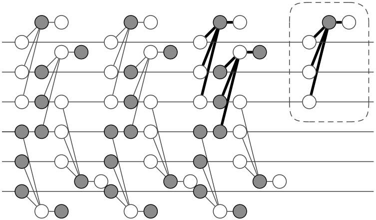
KITAEV CHAIN WITH THREE-BODY INTERACTION

$$H(t) = \sum_{i=1}^{N-2} J_i^{\alpha_i}(t)\sigma_i^{\alpha_i}\sigma_{i+1}^{\alpha_i}\sigma_{i+2}^{\alpha_i}, \tag{15}$$

for $\alpha_i \neq \alpha_{i+1}$ and the restriction that $\alpha_i \in \{x, z\}$, $\alpha_i \in \{x, y\}$ or $\alpha_i \in \{y, z\}$ for all i (i.e. only two different types of Pauli-α occur).



The constant depth square for 6 qubits.



The compression of one Pauli-ZZZ gadget (highlighted) in ZX-calculus for 6 qubits.

SUMMARY AND OUTLOOK

- Usefulness of diagramming tools (ZX-calculus) to show circuit compression
- Re-derived constant-depth properties for several (Ising-, Kitaev-, XY-, TFXY-, TFIM-) models
- Showed constant-depth properties for three-spin interaction Kitaev model
- Indications for constant-depth properties for three-spin transverse-field Ising model

Thanks for your attention!

Hamiltonian Simulation

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HAMILTONIAN SIMULATION

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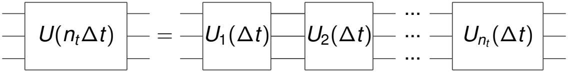
$$U(t_0, t_1) = \tau \exp\left(-i \int_{t_0}^{t_1} H(t)dt\right), \quad H(t) = \sum_{\ell} H_\ell(t) \tag{16}$$

By discretization in time and Trotter decomposition:
Approximate time evolution operator by

$$U(n_t \Delta t) = \prod_{k=0}^{n_t-1} U_{n_t-k}(\Delta t), \quad U_k(\Delta t) = \prod_{\ell} \exp(-iH_{\ell,k}\Delta t). \tag{17}$$

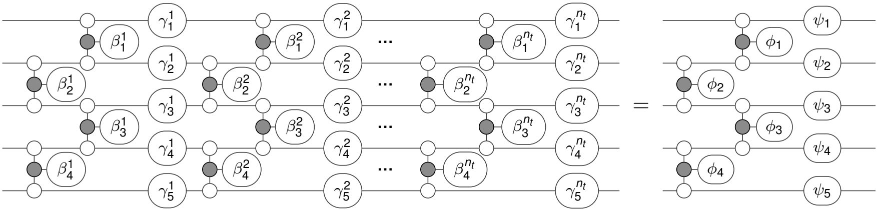


[5] Camps, D. et al.: An Algebraic Quantum Circuit Compression Algorithm for Hamiltonian Simulation. SIAM Journal on Matrix Analysis and Applications 43 (3), pp. 1084–1108, 2022, https://doi.org/10.1137%F21m1439298.

CLASSICAL ISING MODEL

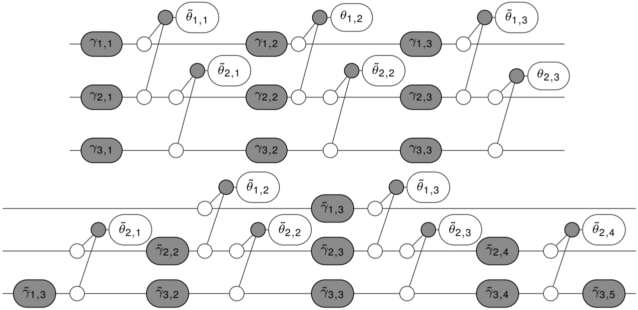
$$H(t) = \sum_{i=1}^{n-1} J_i^{\alpha}(t)\sigma_i^{\alpha}\sigma_{i+1}^{\alpha} + \sum_{i=1}^{n} h_i^{\alpha}(t)\sigma_i^{\alpha}, \quad \alpha \in \{x, y, z\} \tag{18}$$

Approximate time evolution operator for $\alpha = z$:



TRANSVERSE FIELD ISING MODEL

$$H(t) = \sum_{i=1}^{N-1} J_i^{\alpha}(t)\sigma_i^{\alpha}\sigma_{i+1}^{\alpha} + \sum_{i=1}^{N} h_i^{\beta}(t)\sigma_i^{\beta} \tag{19}$$



TRANSVERSE FIELD ISING MODEL WITH THREE-BODY INTERACTION

$$H(t) = \sum_{i=1}^{N-2} J_i^z(t)\sigma_i^z \sigma_{i+1}^z \sigma_{i+2}^z + \sum_{i=1}^{N} h_i^x(t)\sigma_i^x \qquad (20)$$